

# ABSTRACTS OF PAPERS DEPOSITED AT VINITI\*

## CALCULATION OF NUCLEATE BOILING CRISIS OF HELIUM-1 IN INCLINED CHANNELS

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UDC 536.423

The design and thermal calculation of electrical engineering devices thermostatically controlled by liquid helium and having inclined cooled channels requires a knowledge of the first critical heat flux  $q_{cr1}$ . The presently known relationships for calculation of the first critical heat flux in pool boiling of liquids do not include the effect of orientation of the heating surface. At the same time, the available experimental  $q_{cr1}$  data for the boiling of helium-1 show that alteration of the orientation of the heating surface leads to a change in  $q_{cr1}$  by a factor of up to seven [1].

The hydrodynamic model of the nucleate boiling crisis of liquids on inclined surfaces [2] takes into account the effect of orientation and relates it to the change in shape of the vapor bubbles. This model was used to obtain a relationship for calculation of  $q_{cr1}$  in pool boiling of various liquids [2].

It is obvious that orientation of the heating surface will have a pronounced effect on the nucleate boiling crisis in the case of natural circulation in channels. Published results of experimental investigations (by James et al., Sato and Ogata, Keilin et al.) of the effect of orientation of the heating surface of an inclined channel on the first critical heat flux in the boiling of helium confirm this. The nature of the experimental data indicates that the model proposed in [2] can be used for inclined channels.

Adopting the model referred to, we analyzed experimental data for the first critical heat flux in the boiling of helium in rectangular channels of different dimensions with variable orientation of the heat-transfer surface. We obtained the theoretical relationship

$$q_{cr1} = 0.065 (190 - \varphi)^{0.65} L [g\sigma(\rho_l - \rho_v) \rho_v^2]^{0.25} \delta^{0.4} \text{ W/m}^2,$$

which gives the majority of the experimental points for angles of orientation of the heating surface of the channel  $\varphi = 30-150^\circ$ , channel widths  $\delta = (0.3-2) \cdot 10^{-3}$  m, and  $x/\delta = 2.5-55$ , with an error not exceeding  $\pm 40\%$ .

### NOTATION

$\varphi$ , angle between normal to heating surface and gravity vector;  $L$ , heat of vaporization;  $g$ , gravitational acceleration;  $\rho_v, \rho_l$ , density of vapor and liquid;  $x$ , distance from channel entrance;  $\sigma$ , surface-tension coefficient.

### LITERATURE CITED

1. D. N. Lyon, *Int. Adv. Cryog. Eng.*, **10**, 371 (1965).
2. I. P. Vishnev, *Inzh.-Fiz. Zh.*, **24**, No. 1, 59 (1973); *Khim. Neft. Mashinostr.*, No. 3, 16 (1973).  
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DETERMINATION OF HEAT-TRANSFER COEFFICIENT IN CHANNELS  
OF RADIATING NOZZLES

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The paper deals with a flow of gas in a round channel whose walls are heated nonuniformly. The length of the channel is  $l$  and the radius of the transverse flow is  $R$ . The gas moves due to the difference in pressures in the initial (inlet) section and the outlet section ( $\Delta p$ ). This pressure difference is fairly small, the gas velocity is low, and, hence, the Péclet number is less than unity. The gas is assumed to be viscous and incompressible with density  $\rho = \text{const}$ . The flow is assumed to be laminar and the process steady and stabilized, since the stabilization region is small in comparison with the channel length and does not cause significant changes in the considered process.

The heat-transfer equation in a cylindrical system of coordinates has the form [1, 2]

$$\frac{\Delta p}{4\mu l} R^2 \left(1 - \frac{r^2}{R^2}\right) \frac{\partial T}{\partial z} = a \frac{1}{r} \cdot \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right), \quad (1)$$

where  $\mu$  is the dynamic viscosity,  $a$  is the thermal diffusivity, and  $T$  is the gas temperature.

We consider the boundary conditions

$$T(R, z) = T_w(z) \text{ when } z \geq 0, \quad (2)$$

$$T(0, z) \text{ is bounded or } \frac{\partial T(0, z)}{\partial r} = 0, \quad (3)$$

$$T(r, 0) = T_w(0). \quad (4)$$

We solve Eq. (1) by the averaging method, replacing  $\partial T/\partial z$  by

$$f(z) = \frac{2}{R^2} \int_0^R r \frac{\partial T}{\partial z} dr. \quad (5)$$

The solution of Eq. (1) satisfying conditions (2)-(4), with (5) taken into account, has the form

$$T = T_w(z) - \frac{\varphi}{4} f(z) R^2 \left[ \frac{3}{4} - \left(\frac{r}{R}\right)^2 + \frac{1}{4} \left(\frac{r}{R}\right)^4 \right], \quad (6)$$

$$f(z) = \frac{12}{\varphi R^2} \exp\left(-\frac{12}{\varphi R^2} z\right) \int_0^z \frac{dT_w(z)}{dz} \exp\left(\frac{12}{\varphi R^2} z\right) dz, \quad (7)$$

where  $\varphi = \Delta p R^2 / 4l\mu a$ .

It should be noted that if the temperature distribution along the channel wall is linear,  $\partial T_w(z)/\partial z = \text{const}$ , the solution of (6) obtained by the averaging method becomes the exact solution of Eq. (1). Formulas (6) and (7) give the temperature distribution in the channel when the wall temperature has an arbitrary distribution and the gas in the initial section has a constant temperature, equal to the wall temperature in this section. Assuming the characteristic temperature of the wall and gas equal to  $T_w - T_m$ , where  $T_m$  is the mass-average temperature of the gas, we obtain the heat-transfer coefficient  $\alpha = 24/11 \cdot \lambda/R$ , and the corresponding Nusselt number  $Nu = 2.2 \cdot R/\lambda = 48/11$ . In the considered case we found that  $Nu$  is a constant for any temperature distribution  $T_w(z)$  on the wall.

Solutions of corresponding problems, differing from this one in the conditions in the inlet section of the channel, are given in [2, 3]. In view of this the obtained  $Nu$  is in good agreement with the results given in [3].

The proposed method of calculation is fairly simple and can be used to find the required value of  $Nu$  without determination of the eigenvalues and eigenfunctions of the problems dealt with in [2, 3].

LITERATURE CITED

1. L. D. Landau and E. M. Lifshits, *Mechanics of Continuous Media* [in Russian], Moscow (1954).
2. A. A. Gukhman, *Application of Similarity Theory to the Study of Heat- and Mass-Transfer Processes* [in Russian], Vysshaya Shkola, Moscow (1967).
3. B. S. Petukhov, *Heat Transfer and Resistance in Laminar Flow of a Liquid in Tubes* [in Russian], Énergiya, Moscow (1967).

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A BLOCKED FLUIDIZED BED AS A SYSTEM FOR SEPARATING BODIES ACCORDING TO DENSITY

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To stabilize the physicommechanical properties of the solid phase in a fluidized bed and to create conditions for separate withdrawal of constituents of the mixture being separated without the introduction of parts of the unloading device into the bed, the grains of weighting material are connected to the bottom of the tank. The grains are in the form of thin disks and are in groups which are connected (blocked) by flexible threads into chains attached to the air-distributing screen.

In the assembly of chains buoyed by the airstream (blocked fluidized bed), as in a granular fluidized bed, there is a drop in static pressure over the height, and the grains, despite the blocking, have a high degree of mobility. When a pressure drop is present in a medium or system with weakened bonds, bodies are distributed at different levels in it according to their density [1].

A gravimetric device was used to determine experimentally the effective repulsive force  $P_e$  acting on a sphere situated in a blocked fluidized bed, and then the effective density of the bed was calculated:

$$\rho_e = P_e / gQ, \tag{1}$$

where  $g$  is the gravitational acceleration and  $Q$  is the volume of the sphere.

The bed was composed of chains 100 mm long in a  $0.35 \cdot 0.36 \text{ m}^2$  tank. The disks were spaced in the chains at intervals  $t=8, 12, \text{ and } 16 \text{ mm}$ , and were 8 mm in diameter and 1.5 mm thick. The density of the chains on the screen was characterized by the voidage  $\varphi_0$  — the relative free section of the bed during projection onto the air-distributing screen. The voidage in the experiments was 0.07–0.68. The velocity of the inflow was  $V=1.7\text{--}10 \text{ m/sec}$ . The diameter of the immersed spheres was  $d=30\text{--}90 \text{ mm}$ .

The effective density of the bed was given by the empirical relation

$$\rho_e = C \exp \frac{n}{d}, \tag{2}$$

where

$$C = C_1 V^{1.8} \varphi_0^{k_1} t^{-C_2 \varphi_0^{-k_2}}. \tag{3}$$

The values of  $C_1, C_2, k_1,$  and  $k_2$  depended on the depth of immersion  $h_c$  of the center of the sphere in the bed:

$h_c$	$C_1$	$C_2$	$k_1$	$k_2$
$3.5 \cdot 10^{-2}$	3.300	0.60	4.00	0.906
$5.5 \cdot 10^{-2}$	0.204	0.98	0.54	0.400

The form of function  $n$  depends on  $h_c$ . For  $h_c=35 \text{ mm}, n = \ln(\varphi_0/0.215)/(41.2 - 650t)$ .

An index of the good separating ability of the bed is the stability of the effective density over its depth and for bodies of different size. It was established that the separating ability increases with saturation of the bed by components of the aerodynamic drag (Fig. 1).

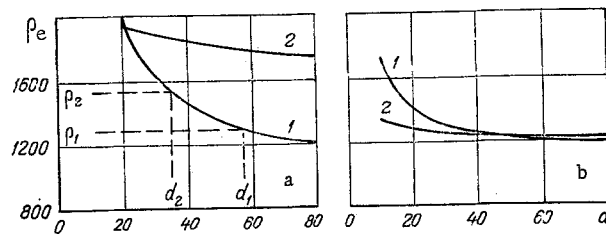


Fig. 1. Effective density of blocked fluidized bed as function of diameter of immersed sphere: a)  $t=8 \text{ mm}; \varphi_0=0.36; V=4 \text{ m/sec}$ ; b)  $t=8 \text{ mm}; \varphi_0=0.25; V=1.92 \text{ m/sec}$ . 1)  $h_c=35 \text{ mm}$ ; 2)  $h_c=55 \text{ mm}$ ;  $\rho_e, \text{ kg/m}^3$ ;  $d, \text{ mm}$ .

The stability of the effective density of the bed in relation to bodies of different size was assessed from the "equifalling" coefficient  $E$  [2]. In the case of a blocked fluidized bed, bodies of different sizes and density with their centers at the same depth in the bed can be regarded as "equifalling". The formula

$$E = \frac{d_1}{d_2} = \ln \frac{\rho_2}{C} / \ln \frac{\rho_1}{C}.$$

was obtained from (2).

In a bed with parameters corresponding to Fig. 1b,  $E = 11.7$ , whereas in an air stream, according to [2],  $E = 1.73$  ( $d_2 = 30$  mm,  $d_1 = 90$  mm).

#### LITERATURE CITED

1. L. G. Loitsyanskii, *Mechanics of Liquids and Gas* [in Russian], Nauka, Moscow (1970), p. 103.
2. P. V. Lyashchenko, *Gravitational Methods of Enrichment* [in Russian], Gostoptekhizdat (1940).

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#### ANALYSIS OF SEPARATION OF DILUTE SUSPENSIONS IN GRANULAR FILTERS

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In the preparation of water from surface sources for industrial- and drinking-water supplies, granular filters are used almost universally for the clarification process. Several techniques are known for intensification of the filtration process, each one capable of increasing the installation efficiency by a factor of 1.5 to 3. Combinations of those techniques are more efficient, but are held back by lack of a suitable method for the analysis of filters (the technological modeling method of Mints [1] makes it possible to regulate the process only by sequential variation of one of the following parameters: the filtration rate or the coarseness and inhomogeneity of the granular layer).

A hydraulic model of the process is proposed on the basis of an analysis of the existing physical notions concerning it. Analysis of the model has made it possible to obtain, in general form, a criterial dependence of the main parameters of the process (velocity of the sorption front, rate of buildup of the losses of head of the silt-infused granular layer, length of the initial section of formation of the concentration front) on the properties of the granular layer and the filtration regime. The specific form of the dependence is obtained on the basis of a battery of experimental studies with granular layers of crushed keramzit (porous clay) [2], quartz sand [1], and scorched rock [3]. As a result, it is made possible to perform calculations for static optimization of water-purification filters with allowance for a combination of standard operations-intensification techniques.

#### LITERATURE CITED

1. D. M. Mints, *Theoretical Fundamentals of Water-Purification Technology* [in Russian], Stroiizdat, Moscow (1964).
2. R. I. Ayukaev, Author's Abstract of Candidate's Dissertation, Inzh.-Stroit. Inst., Kuibyshev (1968).
3. N. D. Artemenok, Author's Abstract of Candidate's Dissertation, Inzh.-Stroit. Inst., Novosibirsk (1970).

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The existing equation for the filtering of aerosols with gradual stoppage of the pores is adequate only for  $dk/d\tau=0$ .

It has been determined that  $dk/d\tau \neq 0$  and  $K=K_i \exp(-m\tau)$  in the filtering of polydisperse aerosols with  $W=const$ .

Adopting a capillary model of the layer, using the fact that for a transmission  $Wd\tau$  of dust and gas flow the quantity of precipitates deposited at the capillary walls is  $(x_i - x)Wd\tau$ , while the thickness of the layer in each capillary increases by  $dr$ , and invoking the obvious relation  $(x_i - x)/x_i = 1 - k$ , we have

$$x_i [1 - K_i \exp(-m\tau)] Wd\tau = -N 2\pi r l dr. \tag{1}$$

After suitable transformations we obtain

$$\left(\frac{1}{Eu}\right)^{1/2} = \left(\frac{1}{Eu_i}\right)^{1/2} - C Re^{1/2} (Ho + \pi_1), \tag{2}$$

where

$$Eu = \frac{\Delta p}{\rho W^2}; \quad Eu_i = \frac{\Delta p_i}{\rho W^2}; \quad C = \frac{x_i}{\pi l} \left(\frac{B\mu d}{N}\right)^{1/2}; \quad Re = \frac{Wd\rho}{\mu};$$

$$Ho = \frac{W\tau}{d}; \quad \pi_1 = \frac{WK_i \exp(-m\tau)}{dm}$$

are dimensionless groups characterizing the hydrodynamic similarity of the process.

The geometrical interpretation of (2) in coordinates  $(Ho_M + \pi_1), [(1/Eu_i)^{1/2} - (1/Eu)^{1/2}]$  is a straight line through the origin. Consequently, the experimental line in the given coordinates is a measure of the filtering process for  $dK/d\tau \neq 0$ .

For engineering calculations it is practical to use the nomogram shown in Fig. 1, which is plotted from raw data spanning a wide range of actual variation of the quantities entering into (2).

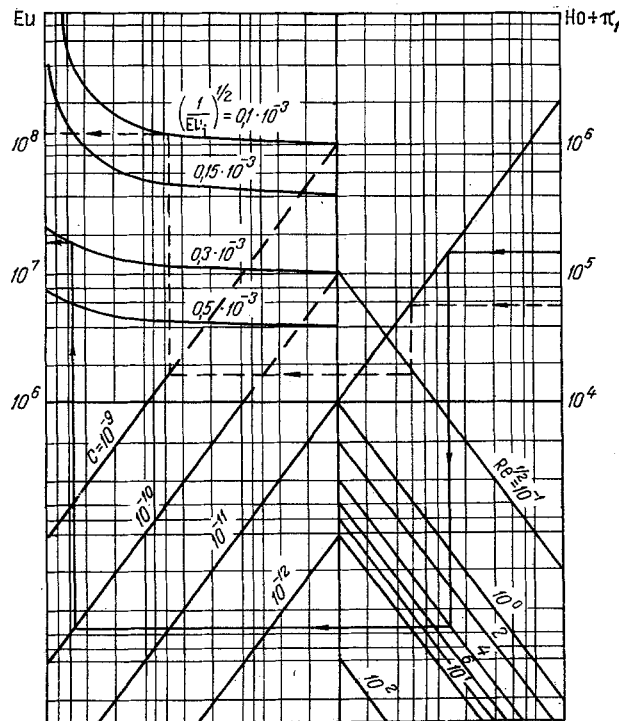


Fig. 1. Nomogram for the determination of  $Eu = \varphi(Eu_i, Ho_M, Re_M, C, \pi_1)$ .

The relative error in the experimental determination of  $E_u$  and in calculation according to the nomogram does not exceed 6%.

The results obtained here have been used to analyze a filter for the purification of exhaust gases from drying equipment.

#### NOTATION

$K$ , flowthrough past filter barrier;  $d$ , particle size of layer, m;  $E_u$ , Euler criterion;  $l$ , length of capillary, m;  $m$ , proportionality factor;  $N$ , number of capillaries per  $1 \text{ m}^2$  filtering surface,  $\text{m}^{-2}$ ;  $\Delta p$ , pressure difference,  $\text{N/m}^2$ ;  $r$ , capillary radius, m;  $W$ , filtering rate, m/sec;  $x$ , ratio of volume of solid-dispersed phase in dust-gas flow to volume of transmitted gas;  $\mu$ , dynamic viscosity coefficient,  $\text{N} \cdot \text{sec}/\text{m}^2$ ;  $\tau$ , filtering time, sec. Subscripts:  $i$ , initial;  $M$ , model.

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### EXPERIMENTAL CORROBORATION OF THE POSSIBILITY OF NEGATIVE-DIRECTED EVAPORATION OF WATER FROM SOIL WITH A DRIED-OUT LAYER

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Experiments have been carried out to reproduce under laboratory conditions the negative-directed evaporation of water from soil, an effect whose possibility under natural conditions has been demonstrated in [1]. The negative-directed evaporation effect is explained by the fact that in the hygroscopic zone of the soil, which has become dried out during the warming stage (during the first half of the day), the content of absorbed moisture is restored during the period of decline of the soil temperature (during the second half of the day) through absorption, not only of water vapor arriving from the depth of the soil, but also of vapor from the ground air [2, 3].

The variation of the evaporation rate from soil is determined from the weight variation of the soil in equal time intervals as the temperature varies. The experimental arrangement consists of three basic components: 1) a sensor that converts soil weight change into an electrical signal; 2) an electronic single-point potentiometer functioning as a differentiating network; 3) a six-point electronic potentiometer (type ÉPP-09), on whose tape is plotted the evaporation rate and variation of the soil temperature, which follows insofar as possible the temperature variation in nature. The experimental procedure is as follows. Air-dried soil is packed into a metal can, which is placed on the dish of an analytical scale. The temperature of the soil is varied according to a prescribed program by varying the intensity of a heat lamp placed over the soil can; the program is scheduled so that the resulting weight variation of the soil does not cause the scale pointer to deflect beyond the limits of the milligram scale. The scale reading is converted by means of a phototransistor into an electrical signal, which is sent to the bridge of the single-point potentiometer. During the inversion cycle of the brushes of the pickup for the ÉPP-09 signals, the bridge acquires an imbalance due to the evaporation-induced variation of the soil weight. The imbalance of the bridge of the differentiating network during equal time intervals is then recorded on the potentiometer tape as a curve tracing the rate of evaporation from the soil.

In the part of the graph corresponding to the period of decline of the soil temperature, the evaporation-rate curve falls below the time (horizontal) axis, thereby indicating negative-directed evaporation due to the absorption of vapor by the soil from the surrounding air. The rate of vapor absorption by the soil in the experiment attains a maximum value  $E_{\text{max}} = -4.3 \cdot 10^{-6} \text{ g/cm}^2 \cdot \text{sec} = -0.15 \text{ mm/h}$  during the period of most rapid decline of the soil temperature.

The experimental results confirm the conclusion in [1] that the effect of negative-directed evaporation from soil with a dried-out layer is possible for a representative daily variation of the soil temperature and ambient air unsaturated with moisture. Ground-air vapor associated with negative-directed evaporation under natural conditions during the afternoon hours of the day is absorbed in the hygroscopic zone of the soil. The negative-directed evaporation effect does not develop into vapor condensation in the soil.

## LITERATURE CITED

1. Zh.-D. Dandaron, Meteorol. Gidrol., No. 7 (1974).
2. Zh.-D. Dandaron, Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1971).
3. Zh.-D. Dandaron, Inzh.-Fiz. Zh., 21, No. 6 (1971).

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OPTIMIZATION OF DRYING PROCESS IN CONVECTIVE DRIERS  
WITH CROSS-INJECTION OF DRYING AGENT

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In the case of driers that use cross-injection of the drying agent it is legitimate to pose the problem of optimizing, in a certain sense, the distribution of the drying-agent parameters over the length of the drier. The solution of the optimal problem is demonstrated in the example of a conveyer-type drier designed for the drying of ceramic blocks. The minimum expenditure of natural gas is taken as the optimality criterion. The mathematical drier model necessary for solving the optimal problem is obtained from the heat and material (with respect to moisture) balance equations for the material and the drying agent, with regard for the assumptions that: 1) the temperature and moisture content of the material and drying agent are distributed over the length of the drier and are lumped in its cross section; 2) the drying rate is proportional to the moisture content of the material, where the proportionality factor (drying coefficient) depends on the composition and dimensions of the blocks and on the parameters (velocity, temperature, and moisture content) of the agent; 3) the heat content of the material is a linear function of its temperature; 4) heat transfer from the drying agent to the material is realized by convection. The outcome is a mathematical model of the drier in the form of a system of two nonlinear ordinary differential and two nonlinear algebraic equations. The adequacy of the model as representing the real object has been tested by means of experimental studies at the Kuchinsk Combine for Ceramic Lining Materials; the relative deviation of the analytical and experimental data does not exceed 6%. The optimal problem is solved with regard for the constraints imposed on the moisture content of the blocks as they emerge from the drier, namely a certain allowed maximum determined by technological specifications, along with constraints on the rate of change of the moisture content of the blocks along the length of the drier, so that it must not exceed the drying rate determined from the equation for the safety-limit drying curve. The controlling factor is the distribution of the volumetric flow of natural gas along the length of the drier. The following algorithm is used to solve the above-stated optimal problem: 1) The constraints are taken into account by the method of penalty functions; 2) the relations dictated by the algebraic equations are satisfied by selecting the appropriate values for the temperature ( $t_g$ ) and moisture content ( $d$ ) of the drying agent, for which the algebraic equations are solved by the Newtonian method for  $t_g$  and  $d$ ; 3) Pontryagin's maximum principle is then applied and expressions are obtained for the function  $H$  and auxiliary variables  $\psi_1$  and  $\psi_2$ . As a result of solving the equations for  $\psi_1$  and  $\psi_2$  it is inferred that  $\psi_2=0$  and that  $\psi_1$  is expressed in terms of integrals that are evaluated by numerical methods; 4) the gradient approach is used to minimize the function  $H$ . The program used to implement the foregoing algorithm and its corresponding numerical procedure is written in ALGOL and FORTRAN for computers of the Minsk and BESM series. The distribution curves obtained for the volumetric flow of natural gas along the length of the drier in the experimental studies described above and by calculations according to the indicated algorithm for identical values of the input parameters of the material are compared with one another.

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The drying of green malt in a thin layer is investigated; the results indicate that the malt drying rate under these conditions is practically independent of the velocity of the heat-transfer agent, while the temperature of the drying agent has a considerable influence. The actual process takes place in the second drying period.

A graph-analytical method is devised on the basis of the investigations for determining the moisture conductivity of the malt. The method essentially entails representing the drying curve by a set of exponential functions, the sum of which can be expressed by the following equation under certain simplifying assumptions [1]:

$$\frac{W - W_{eq}}{W_0 - W_{eq}} = \sum_{n=1}^{\infty} B_n \exp\left(-\mu^2 \frac{k\tau}{R_e^2}\right). \quad (1)$$

The drying curves are represented in semilog coordinates  $\log \frac{W - W_{eq}}{W_0 - W_{eq}} = f(\tau)$ .

The approximate values of the moisture-conductivity coefficients of the malt for various temperatures of the drying agent are determined from the slope of the curve segment having the least slope relative to the  $\tau$  axis:

$$\operatorname{tg} \alpha = C \frac{\mu_1^2 k}{R_e^2}, \operatorname{sec}^{-1}, \quad (2)$$

$$k = \frac{R_e^2 \operatorname{tg} \alpha}{\mu_1^2}, \operatorname{m}^2/\operatorname{sec}. \quad (3)$$

The values of the moisture-conductivity coefficients of green malt are determined more precisely by numerical evaluations of Eq. (1) on a digital computer for a specified range of temperatures and comparison of the experimental and computed values of the moisture content of the malt.

The temperature dependence of  $k$  is found to be

$$k = AT_d^m, \operatorname{m}^2/\operatorname{sec}. \quad (4)$$

This relation makes it possible to calculate the value of  $k$  with sufficient accuracy.

#### NOTATION

$T_d$ , temperature of drying agent, °K;  $W$ ,  $W_0$ ,  $W_{eq}$ , instantaneous, initial, and equilibrium moisture contents of material, %;  $\mu_n = \pi n$ , roots of the Bessel function of the first kind of order zero;  $B_n = 6/\mu_n^2$ , dimensionless constant;  $k$ , moisture-conductivity coefficient,  $\operatorname{m}^2/\operatorname{sec}$ ;  $\tau$ , time, sec;  $R_e$ , equivalent grain radius, equal to 0.0028 m;  $\beta$ , moisture-transfer coefficient, m/sec;  $A$ , constant equal to  $3.981 \cdot 10^{-46}$ ;  $m$ , power exponent, equal to 14;  $n$ , natural number series.

#### LITERATURE CITED

1. A. V. Lykov, Theory of Drying [in Russian], Gosénergoizdat, Moscow (1950).  
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HEAT AND MASS TRANSFER, STRAINS, AND STRESSES  
ASSOCIATED WITH FREEZING IN A LAYER OF FINELY  
DISPERSE WET SOIL

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UDC 624.131:551.34

The freezing of wet finely disperse soils comprises a complex physical process, which is accompanied by heat and mass transfer as well as the development of stresses and strains in the thawed and frozen soil zones. It is generally known that the interstitial water of the thawed zone in the freezing of finely disperse soils can migrate toward the freezing front. This process has the effect of dehydrating and compacting the thawed zone. On the other hand, the interstitial water expands during freezing, separating the particles of the soil matrix. The sum total of these processes governs the development of stresses and strains, which can attain very high values and present a hazard to engineering installations.

The one-dimensional problem of freezing of a volume of wet soil bounded below by a layer of permafrost and above by structural elements of an installation is analyzed in order to derive the fundamental equations characterizing the process. The migration of moisture in the thawed zone is described by the diffusion equation for a porous medium with a deformable matrix. The relationship between the stresses in the matrix of the thawed soil and the porosity is formulated within the scope of theory of hereditary creep.

The motion of the interface between the thawed and frozen zones is described by an equation deduced from the heat-balance equation; the evolution of the porosity and moisture fields is related to the quantity of water transferred into the solid phase; the phase-transition heat released in this case is related to the jump of the heat flux at the interface.

The end result is a system of eight equations describing the process in question. An approximative method is proposed for solving this system by reduction to a system of three first-order ordinary differential equations and a series of finite relations; a numerical example is given. The results of the calculations show that the freezing-induced stresses under the given conditions attain 4 or 5 kg/cm<sup>2</sup>, which can produce sizable loads on engineering installations. Experiments give results of the same order.

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HEAT CAPACITY OF A MULTICOMPONENT HETEROGENEOUS  
SYSTEM WITH A CONCENTRATION MATRIX OF MAXIMUM RANK

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UDC 536.722

An analytical description is given for the heat capacity  $C_Z$  ( $z=p$  or  $V$ ) of multicomponent heterogeneous systems on the basis of phenomenological thermodynamics. The heat capacity  $C_Z$  is interpreted, as is customary in the literature, to mean  $C_p$  or  $C_V$  for constant masses  $M_1, M_2, \dots, M_n$  of the system components ( $n$  is the number of components) and constant pressure  $p$  or volume  $V$  of the system. The thermodynamic system, treated in accordance with the conditions of the definition, is specified by a set of external parameters  $S, z, M_1, M_2, \dots, M_n$ , where all the parameters except the first (the entropy) are fixed and the entropy can be freely varied. In accordance with the theory of multicomponent heterogeneous systems [1], the complete system of equations in differential form, uniquely expressing the dependence of all internal parameters of the given thermodynamic system on its external conditions, has the form

$$d\mu_{i1}(T, z_j^*, x_{11}, \dots, x_{n1}) = d\mu_{ir}(T, z_j^*, x_{1r}, \dots, x_{nr}) \quad (i = 1, 2, \dots, n), \quad (1)$$

$$\sum_{j=1}^r d[m_{js}j(T, z_j^*, x_{1j}, \dots, x_{nj})] = dS. \quad (2)$$

$$\sum_{j=1}^r d(m_j x_{ij}) = 0 \quad (i = 1, 2, \dots, n), \quad (3)$$

$$\sum_{i=1}^n dx_{ij} = 0 \quad (j = 1, 2, \dots, r), \quad (4)$$

$$dp_j(T, v_j, x_{1j}, \dots, x_{nj}) = 0 \quad (j = 1, 2, \dots, r) \text{ for } z = p \quad (5)$$

or

$$\sum_{j=1}^r d(m_j v_j(T, p, x_{1j}, \dots, x_{nj})) = 0 \text{ for } z = V, \quad (5')$$

where  $T$  is the temperature,  $z_j^*$  is the internal parameter of the  $j$ -th phase, conjugate with  $z_j$  ( $v_j$  or  $p$ ),  $x_{ij}$  is the concentration of the  $i$ -th component in the  $j$ -th phase,  $m_j$  is the mass of the  $j$ -th phase,  $s_j$  and  $v_j$  are the specific enthalpies and volume of the  $j$ -th phase, and  $r$  is the number of phases of the thermodynamic system.

Thermodynamic systems determined by a set of external parameters  $S, z, M_1, M_2, \dots, M_n$ , of which all but the entropy  $S$  are fixed, have all uniquely determined (fixed) internal intensive parameters in the case where the number of phases  $r = n + 1$  and  $r = n + 2$  for  $z = p$  and the number of phases  $r = n + 2$  for  $z = V$  [1]. A change of state of the indicated thermodynamic systems (shift of equilibrium) takes place under isothermal conditions, and the heat capacity  $C_z$  is infinite.

The heat capacity of the thermodynamic systems for  $r \leq n, z = p$ , and for  $r \leq n + 1, z = V$ , is given by the expression

$$C_z = T \frac{dS}{dT} = T \sum_{j=1}^r m_j \frac{\partial s_j}{\partial T} + m_j \frac{\partial s_j}{\partial z_j^*} \frac{dz_j^*}{dT} + \sum_{i=1}^n m_j \frac{\partial s_j}{\partial x_{ij}} \frac{dx_{ij}}{dT} + s_j \frac{dm_j}{dT}. \quad (6)$$

Solving the system of linear equations (1)-(5) or the system (1)-(4), (5') for each of the variables  $dT, dv_j, dx_{ij}, dm_j$  ( $i = 1, 2, \dots, n; j = 1, 2, \dots, r$ ) or for the variables of the second system  $dT, dp, dx_{ij}, dm_j$  ( $i = 1, 2, \dots, n; j = 1, 2, \dots, r$ ), we readily obtain paired relations, i.e., the total derivatives of the temperature with respect to the internal parameters in the heat-capacity expression (6).

The expression obtained for the heat capacity of multicomponent heterogeneous systems is a useful analytical expression for the description and analysis of the thermal properties of systems. The given analytical description of the heat capacity can be used to arrive in analogous fashion at an analytical description of other second-order derivatives of the thermodynamic functions of multicomponent heterogeneous systems.

#### LITERATURE CITED

1. L. S. Palatnik and A. I. Landau, Phase Equilibria in Multicomponent Systems [in Russian], KhU, Khar'kov (1961).

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Original paper submitted March 9, 1977.

#### ESTIMATING SOLUTION CONVERGENCE OF A NONLINEAR HEAT-TRANSFER PROBLEM TRANSFORMED BY THE SMALL-PARAMETER METHOD

G. B. Cherepennikov

UDC 536.2.01

We consider a nonlinear problem in heat transfer between an unbounded cylinder ( $0 \leq r \leq R$ ) and a medium at constant temperature  $T_c$  with a heat-transfer coefficient  $\alpha$  which depends on the surface temperature of the body.

The source of nonlinearity and the desired solution are represented in the form of series in powers of the small parameter  $\beta$  ( $\beta < 1$ ):

$$\alpha(T(R, \tau)) = \sum_{n=0}^{\infty} \alpha_n [\beta T_n(R, \tau)]^n, \quad T(r, \tau) = \sum_{m=0}^{\infty} \beta^m T_m(r, \tau),$$

and the original problem is reduced to a system of linear equations and boundary conditions whose solution can be written as

$$T(r, \tau) = LT_{in} + LT_b = L \left[ \sum_{m=0}^{\infty} T_{c(m)}(\tau) \beta^m \right], \quad (1)$$

where L is the symbol of the operator used;  $T_{in}$  is the initial condition (in the present case  $T_{in} = 0$ );  $T_b$  is the boundary condition;  $T_{c(m)}(\tau)$  represents the sums of the terms of the expansion which depend on the previously determined  $T_k(R, \tau)$  ( $k < m$ ) determined in the process of carrying out the procedure of the small-parameter method.

It can be seen that the solution of the problem converges if we have convergence of the series under the operator sign. It is not possible to investigate the convergence of the series, but we have set up a majorizing series for it whose convergence conditions, according to the Weierstrass criterion, are also valid for the series being investigated, and therefore for the desired solution as well.

By means of this approach, it was established that the solution of the problem under consideration converges if

$$-\frac{\alpha_m}{\alpha_{m+1}} \frac{1}{T_c} < \beta < \frac{\alpha_m}{\alpha_{m+1}} \frac{1}{T_c}. \quad (2)$$

In particular, for  $\alpha(T(R, \tau)) = \alpha_0 + \alpha_1 \beta T(R, \tau)$ , the convergence condition takes the form

$$-\frac{\alpha_0}{\alpha_1} \frac{1}{T_c} < \beta < \frac{\alpha_0}{\alpha_1} \frac{1}{T_c}. \quad (3)$$

It can be represented as

$$\left| \frac{\alpha(T(R, \tau)) - \alpha_0}{\alpha_0} \right| < 1, \quad (4)$$

from which it follows that the convergence of the solution is greatly influenced by the coefficient  $\alpha_0$ , i.e., by what temperature is taken as the origin of our calculations. If the "zero" temperature we take is the temperature at which the source of nonlinearity has its maximum value, then  $\alpha_0 = \alpha_{max}$  and, by (4), the convergence condition is automatically satisfied.

The difference  $\Delta\alpha = (\alpha(T(R, \tau)) - \alpha_0)$  is the "increment" of the value of the source of nonlinearity in the range of variation of temperature from  $T(R, 0)$  to  $T(R, \tau)$ . It can be seen that the greater the value of  $\Delta\alpha$ , the higher will be the ratio  $|\Delta\alpha/\alpha_0|$  and the worse will be the convergence of the solution of the problem. Consequently the rate of convergence varies within a given temperature range. In the general case it is fairly high near a temperature close to zero and becomes worse as we approach the boundaries of the interval. This conclusion is confirmed by the results of the calculations we made.

It was noted that for other types of sources of nonlinearity (which depend on the temperature, the thermophysical properties, the internal generation of heat, etc.), when we use the small-parameter method, the convergence conditions are analogous in form. This gives reason to suppose that the conclusions and recommendations of our study are valid beyond the limits of the particular case considered.

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INTEGRAL EQUATIONS OF STEADY-STATE HEAT-CONDUCTION  
 PROBLEMS FOR BODIES WITH CURVILINEAR CUTS

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UDC 539.12

We consider a two-dimensional steady-state problem in heat conduction for a body with curvilinear cuts, at one group of which the temperature is specified, at the second group of which heat fluxes are specified, and at the third group the conditions of thermal permeability are specified. It is assumed that the region occupied by the body is mapped by the function  $z = \omega(\zeta)$  conformally onto the exterior of the unit circle, and conditions of the first or second kind are given on its boundary. The temperature field in the transformed region can be represented by the sum

$$T(\zeta, \bar{\zeta}) = T_n(\zeta, \bar{\zeta}) + T_*(\zeta, \bar{\zeta}),$$

where  $T_n$  is the temperature in the region under consideration without any cuts, satisfying the boundary condition on the contour of the unit circle;  $T_*$  is the temperature due to the presence of the cuts.

The boundary conditions of the problem in the transformed region can be written as follows:

$$\begin{aligned} T_*^\pm(t) &= f^\pm(t) - T_n(t), \quad t \in L_1, \\ \lambda \left( \frac{\partial T_*}{\partial n} \right)^\pm &= \mp |\omega'(t)| q^\pm(t) - \lambda \frac{\partial T_n}{\partial n}, \quad t \in L_2, \\ \lambda \left( \frac{\partial T_*}{\partial n} \right)^\pm - h(t) |\omega'(t)| (T_*^+ - T_*^-) &= |\omega'(t)| q_a(t) - \lambda \frac{\partial T_n}{\partial n}, \quad t \in L_3, \\ T_*|_{\rho=1} &= 0 \quad \text{or} \quad \frac{\partial T_*}{\partial \rho} \Big|_{\rho=1} = 0, \end{aligned} \quad (1)$$

$$(2)$$

where  $\lambda$  is the thermal conductivity of the body,  $h(t)$  is the thermal permeability of the cut;  $n$  is the normal to the left edge of the cut, denoted by the index +;  $f^\pm(t)$ ,  $q^\pm(t)$ ,  $q_a(t)$  are the given temperature and heat fluxes;

$L = \bigcup_{j=1}^3 L_j$  is the preimage of the contour along which the cuts are made in the physical plane.

The perturbed temperature field is to be found in the form

$$\begin{aligned} T_*(\zeta, \bar{\zeta}) &= 2\text{Re } F(\zeta), \quad F(\zeta) = \frac{1}{2\pi i} \int_{L_1} [f_1(t) M_2(t, \zeta) + i\gamma_1(t) M_1(t, \zeta)] dt + \\ &+ \frac{1}{2\pi i} \int_{L_2} [\gamma_2(t) M_2(t, \zeta) - q_2(t) M_3(t, \zeta)] dt + \\ &+ \frac{1}{2\pi i} \int_{L_3} \gamma_3(t) M_2(t, \zeta) dt - \sum_{k=1}^N \frac{c_k}{2\pi i} \int_{L_{1k}} M_3(t, \zeta) dt + \frac{c_0}{2}, \end{aligned} \quad (3)$$

where

$$\begin{aligned} M_j(t, \zeta) &= \frac{1}{t-\zeta} \mp (-1)^j \frac{\bar{\sigma}_\alpha \bar{\zeta}}{i\bar{\zeta}-1}, \quad j = 1, 2; \quad \sigma_\alpha = \exp(i\alpha); \\ M_3(t, \zeta) &= \bar{\sigma}_\alpha \left[ \ln(t-\zeta) \mp \ln(\bar{i}\bar{\zeta}-1) - \frac{1 \mp 1}{2} \ln \zeta \right]; \\ 2f_1(t) &= f^+(t) - f^-(t); \quad 2\lambda q_2(t) = |\omega'(t)| [q^+(t) + q^-(t)]; \end{aligned}$$

$L_1 = \bigcup_{k=1}^N L_{1k}$ ,  $\alpha$  is the angle between the normal  $n$  and the abscissa axis;  $\gamma_j(t)$  are unknown functions bounded at the ends of the cuts;  $c_k$  ( $k = \overline{0, N}$ ) are real constants determined from the conditions of boundedness at the ends of the cuts  $L_{1k}$  of the function  $\gamma_1(t)$  and the condition of boundedness of the temperature field at infinity. In the kernels  $M_j$  the upper sign corresponds to the first boundary condition in (2), the lower sign to the second.

From the boundary conditions (1), taking account of the expression (3), we have obtained a system of singular integral equations for determining the functions  $\gamma_j(t)$ . We have also written out a system of integral equations for the corresponding periodic problems for an infinite body.

The paper gives the solution of the integral equation for a periodic system of arc-shaped cuts at the edges of which an arbitrary temperature is maintained.

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## GENERALIZED VARIABLES OF THE HEAT-TRANSFER PROCESS IN MULTILAYER PLATES WITH HARMONICALLY VARYING TEMPERATURE

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UDC 536.21

The parameter of the heat-transfer process for harmonic variation of temperature is the quantity  $\xi = (a/\omega)^{1/2}$ , in meters, determined by the thermal diffusivity and the cyclic frequency of the oscillation. If the parameter is taken as a characteristic scale, the temperature field of a multilayer plate consisting of  $N$  layers, for boundary conditions of the third kind, will be a function of the following generalized coordinates:

- a) dimensionless time or the phase of the oscillation of the temperature of the external movable medium

$$Fo^* = \frac{a_n \tau}{\xi_n^2},$$

- b) the relative thicknesses of the layers of the plate

$$D_1, \dots, D_n, \dots, D_N, \text{ where } D_n = \frac{\delta_n}{\xi_n},$$

- c) the relative coordinate  $D_x = x/\xi_n$ ,

- d) numbers expressing the ratio of the tangents of the angles of inclination of the tangent lines to the temperature curves at the junctions of the layers and on the surfaces of the plate:  $W_0 = \alpha_{c1}/s_1$ ,  $W_1 = s_1 s_2, \dots$ ,  $W_n = s_n/s_{n+1}, \dots$ ,  $W_{N-1} = s_{N-1}/s_N$ ,  $W_N = s_N/\alpha_{c2}$ .

The last list of quantities can be combined by the notation  $W_n = w_n/w_{n+1}$ ; the subscript after  $W$  indicates the number of the cross section to which the given quantity relates; for the boundary layer  $w = \alpha$ , for a layer of the plate  $w = s$ , where  $s = \lambda/\xi$   $W/m^2 \cdot K$ ;  $s$  has the meaning of thermal conductivity of a unit dimensionless layer. The quantity  $s$  may be defined as the cyclic thermal conductivity, since it serves the function of the proportionality factor  $\lambda$  when we pass to the relative coordinates  $D_x$ :

$$q = \lambda \frac{\partial \theta}{\partial x} = \frac{\lambda}{\xi} \left( \frac{\partial \theta}{\partial \frac{x}{\xi}} \right) = s \frac{\partial \theta}{\partial D_x} \quad W/m^2.$$

The use of the generalized coordinates  $Fo^*$ ,  $D$ , and  $W$  is illustrated by the equations of the temperature field of the plate consisting of  $N$  layers. Such equations can be obtained if for each layer we take the characteristic coordinate axes, and the damping of the temperature oscillations and the phase shift are considered layer by layer. In this case the polynomials of the temperature-field equation for one layer are included as a component of the equation for the next layer. As a result, the computer calculations are formulated according to a recurrent (cyclic) program. Such a program is universal, since it is suitable for plates containing any number of layers. Before beginning the calculations, we introduce  $N$ ,  $Fo^*$ , and  $W_N = s_N/\alpha_{c2}$  into the computer memory; before calculating the next layer  $n$ , we introduce the generalized variables  $D_n$  and  $W_{n-1} = s_{n-1}/s_n$ .

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TAKING ACCOUNT OF THE FINITE VELOCITY  
OF HEAT PROPAGATION

K. V. Lakusta

UDC 517.949.22

In the numerical construction of generalized nonsteady temperature fields in one-dimensional bodies described by the solution of the equation

$$\tau_r c \rho \frac{\partial^2 u}{\partial t^2} + c \rho \frac{\partial u}{\partial t} - \lambda \left( \frac{\partial^2 u}{\partial z^2} + \frac{v}{z} \cdot \frac{\partial u}{\partial z} \right) = f(t, z), \quad (1)$$

$(R_1 \leq z \leq R_2, 0 \leq t < T (T < +\infty))$

with the initial conditions

$$u|_{t=0} = \varphi_1(z); \quad \frac{\partial u}{\partial t} \Big|_{t=0} = \varphi_2(z) \quad (2)$$

and the boundary conditions

$$\left( \Pi_1 \lambda \frac{\partial u}{\partial z} - \Pi_2 u \right) \Big|_{z=R_1} = \psi_1(t); \quad \left( \Pi_3 \lambda \frac{\partial u}{\partial z} + \Pi_4 u \right) \Big|_{z=R_2} = \psi_2(t), \quad (3)$$

we study the problem of taking account of the velocity of heat propagation for the description of thermal processes. Here  $c$  is the specific heat capacity;  $\rho$  is the density;  $\lambda$  is the thermal conductivity;  $\tau_r$  is the relaxation time of the thermal stress;  $f(t, z)$  is the power of the heat sources;  $\varphi_1(z)$  and  $\varphi_2(z)$  are the initial temperature and its velocity of propagation;  $\psi_1(t)$  and  $\psi_2(t)$  are the temperatures on the boundaries of the region;  $\Pi_i$  ( $i = \overline{1, 4}$ ) take on the values 0 and 1, depending on the kind of boundary conditions applicable;  $\nu = 0, 1, 2$ .

Replacing the derivatives with respect to the coordinate and to time with the corresponding difference ratios, we obtain a finite-difference analog of the problem. The algorithm of the computing process is carried out on the basis of an explicit-difference scheme.

Our investigations of the temperature fields constructed in this way showed that taking account of a finite velocity of propagation of a substance on the M-6000 and ES-1020 computers is effective when the relaxation time of the thermal stress is a quantity of the order of  $10^{-2}$ - $10^{-9}$  and  $10^{-2}$ - $10^{-11}$  sec. Therefore, for relaxation times which do not lie within these limits, it is desirable to use the classical heat-conduction equation for describing the thermal processes.

Representing Eq. (1) in dimensionless form by the parameter  $M = c_l/c_q$  ( $c_l$  is the velocity of propagation of the longitudinal wave,  $c_q$  is the velocity of the heat propagation), we carried out a simulation of the problem (1)-(3) for different values of  $M^2$ . The simulation showed that the temperature increases as  $M^2$  decreases.

It was noted that: a) the program we worked out is suitable for calculating dynamic temperature stresses; b) use of the classical heat-conduction equation leads to erroneous results for the case of very small values of time.

The paper includes graphs showing the distribution of dimensionless temperature as a function of a dimensionless geometric variable and of dimensionless time, respectively, for fixed values of time and the coordinate.

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CALCULATION OF THE SOLIDIFICATION PROCESS OF MELTS,  
TAKING ACCOUNT OF SUPERHEATING

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UDC 621.746.6

The existing methods for taking account of superheating in calculations of the solidification of melts assume that the heat of superheating is removed at a separate stage preceding the solidification, or else during the entire time of solidification, proportionally to the mass of the solidifying metal. Both methods make

it possible to determine fairly accurately the total solidification time, but they distort the picture of the formation of a hard crust during the initial period, since they do not take account of the nonuniformity of the removal of the superheating with respect to time.

For a differentiated accounting of the effect of the heat of superheating on the solidification process, we introduce the concept of the intensity of liberation of the heat of superheating:

$$I_{\text{sup}} = \frac{dQ_{\text{sup}}}{dv},$$

where  $dQ_{\text{sup}}$  is the amount of heat of superheating liberated by the melt during the time required to solidify a volume  $dv$ .

Further specifying the linear variation of  $I_{\text{sup}}$  as a function of the volume of the solidified part of the melt, we determine the variation of  $Q_{\text{sup}}$  as a function of  $v$  and make use of this relation in setting up the heat-balance equation for an elementary volume of the solidified melt. As a result, we obtain a solution of the problem of solidification of an unbounded plate and an unbounded cylinder in the form of the function  $Fo = Fo(\delta, Bi)$ , in which the Fourier criterion characterizes the time of solidification, the Biot criterion characterizes the intensity of cooling of the melt, and  $\delta$  characterizes the relative thickness of the solidified part.

For the solution of the equation we obtained for  $\delta$  under conditions of variable intensity of cooling, we propose a graphical-analytic method consisting of the construction of the nomograms for  $\delta = \delta(Fo)$  for different values of  $Bi$  and the step-by-step determination of  $\delta$  from the known relation  $Bi = Bi(Fo)$ .

In carrying out practical calculations to determine the kinetics of solidification of melts, the determination of the relation  $Bi = Bi(Fo)$  is a matter of considerable complexity. In the solidification of a melt in cooled metallic forms, it is much easier to determine the variation of the specific heat flux  $q$  as a function of time. In order to convert  $q = q(t)$  into  $Bi = Bi(Fo)$ , we propose a graphical-analytic method consisting of the following. Simultaneously with the calculations needed for constructing the nomograms for  $\delta = \delta(Bi, Fo)$ , we determine and plot on a graph the relation  $G = G(Fo)$  for different values of  $Bi$ . Here  $G$  is a dimensionless heat flux which represents the ratio of the true specific heat flux to the standard specific heat flux; as the standard, we take the heat flux through the surface of the solidifying melt at the time when the solidification ends ( $\delta = 1$ ), under conditions of infinite intensity of cooling ( $Bi = \infty$ ). The relation  $q = q(t)$ , known from experiment or calculation, is converted into  $G = G(Fo)$  and plotted on the nomogram  $G = G(Fo, Bi)$ . From the points of intersection of this curve with the curves of the nomogram, we determine the relation  $Bi = Bi(Fo)$ , which is used for determining the kinetics of the solidification of the melt. The nomograms  $\delta = \delta(Fo, Bi)$  and  $G = G(Fo, Bi)$  must be constructed for identical solidification conditions. In the study we give examples of the construction of nomograms and the procedure for the calculations. We also give an analysis of the effect of superheating on the kinetics of solidification.

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## PLASMA PARAMETERS OF A GAS-DISCHARGE ION SOURCE

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UDC 533.932

The paper gives the results of an investigation of the parameters of a high-velocity stream of rarefied plasma generated by an ion source with electron bombardment. For stabilization of the discharge in possibly small magnetic fields the source was equipped with an anode diaphragm. The operation of the source was characterized by the following parameters: discharge current 0.3 A, discharge voltage relative to working chamber 100 V, maximum magnetic field at center on source axis  $\sim 65$  Oe.

Technically pure dry argon was used as the working gas. The accelerated ion stream entered the working chamber, in which the residual gas pressure was  $\sim 10^{-6}$  torr. The parameters of the plasma issuing from the source and partially in the discharge chamber of the latter were measured by a movable thermoanemometric

probe with a flat working surface 2.2 mm in diameter. The measurements were made directly on the jet axis at pressures  $p = 5 \cdot 10^{-5}$  and  $1.5 \cdot 10^{-4}$  torr in the working chamber. The pressure in the source discharge chamber in these cases was  $p = 1.5 \cdot 10^{-3}$  and 3.5 torr, respectively.

The electron energy distribution was very close to Maxwellian. The plasma potential  $\varphi_0$  was determined by the second derivative method and also from the electronic part of the current-voltage characteristic plotted on a semilogarithmic scale. The plasma noise collected by the probe was also measured during the experiments and the result was used to check the measurement of the potential. The maximum plasma noise corresponded to the plasma potential. A block scheme for determination of  $d^2I_e/dV^2$  is given. The plasma potential  $\varphi_0$  found from the point  $d^2I_e/dV^2 = 0$  and the noise maximum corresponded better with the onset of deviation of the semi-logarithmic characteristic from linearity than the point of intersection of the asymptotes.

The mass velocity  $U_\infty$  of the stream was evaluated from the temperature characteristic  $T_w = T_w(V)$  of the probe operating as a thermocouple. The method of treating the temperature characteristic is described. The obtained values of the stream velocity  $U_\infty \approx 7.5 \cdot 10^5$  cm/sec agreed satisfactorily with the values of  $U_\infty$  calculated on the assumption that the accelerating potential is equal to the difference between the source anode potential and the local plasma potential. The spread of the obtained values did not exceed  $\pm 7.5\%$ .

The electronic and ionic parts of the probe characteristic were used to determine the charged-particle concentration.

The temperature and concentration of neutrals in the source discharge chamber were determined from the temperature characteristic and the probe energy-balance equation. For pressures in the ionization chamber  $p_i = 1.5 \cdot 10^{-3}$  and  $3.5 \cdot 10^{-3}$  torr,  $T_n = 1570$  and  $1450^\circ\text{K}$  and  $n_n = 9 \cdot 10^{12}$  and  $2.3 \cdot 10^{13}$   $\text{cm}^{-3}$ , respectively.

The presented results show the nature of the variation of the main parameters of the high-velocity stream when the plasma from the discharge chamber expands into the vacuum.

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Original paper submitted May 27, 1975.

## EXPERIMENTAL INVESTIGATION OF X-RAY SCATTERING IN HIGHER FORMATES

K. D. Guseinov and G. A. Aslanov

UDC 536.2

The authors obtained plots of the intensity of monochromatic x rays ( $\lambda_K = 0.71 \text{ \AA}$ ) against the angle for liquid hexyl, heptyl, and octyl formates.

The investigations showed that all the formates give a scattering picture of the same type. The difference in scattering consisted in the absolute value of the intensity, the increase in intensity of the first maximum, and reduction of its half-widths on transition to higher formates. Since the main contribution to the intensity of the first maximum is made by intermolecular scattering, we can conclude that an increase in the length of the molecules leads to an increase in their degree of order.

The mean distance between the nearest molecules can be calculated from the relation  $\bar{R} = 7.73/S_1$  [1]. For formates of normal structure  $S_1 = 1.4 \text{ \AA}^{-1}$  and  $\bar{R} = 5.56 \text{ \AA}$ .

An analysis of the x-ray-intensity curves showed that the investigated formates are similar substances and the law of corresponding states can be applied to this series for correlation of the viscosity data.

The obtained experimental values of viscosity and density [2] of formates at different pressures and temperatures were correlated by the relation  $\Delta\eta/\Delta\eta_1 = f(\rho/\rho_1)$ , where  $\Delta\eta$  is the excess viscosity;  $\Delta\eta_1$  is the difference in viscosity of the liquid at  $P = 100$  bar and  $T = T_b$  and vapor at normal boiling point;  $\rho$  is the density of the liquid;  $\rho_1$  is the density of the liquid at  $P = 100$  bar and the boiling point.



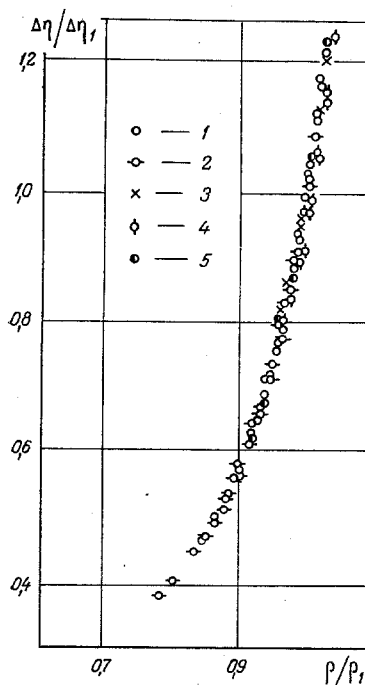


Fig. 1. Plot of viscosity of formates against density in relative coordinates: 1) ethyl formate; 2) butyl formate; 3) amyl formate; 4) hexyl formate; 5) heptyl formate.

The result of this treatment is shown in Fig. 1. The characteristic relation, which was the same for all the substances, had the form

$$\frac{\Delta\eta}{\Delta\eta_1} = \sum_{i=0}^2 y_i \left( \frac{\rho}{\rho_1} \right)^i .$$

where  $y_0=9.07267$ ;  $y_1=-21.77529$ ;  $y_2=13.69620$ .

The scatter of individual points from the averaging curve was less than 2% on the average.

#### LITERATURE CITED

1. A. F. Skryshevskii, Structural Analysis of Liquids [in Russian], Vysshaya Shkola, Moscow (1971).
2. K. D. Guseinov and G. A. Aslanov, Zh. Fiz. Khim., No. 11 (1976).

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